NAME

Welcome to AP Calculus 1!

We are hoping for a wonderful and a very successful year! To that end, we have attached your summer assignment for AP Calculus. It should be completed for the first day of school. Use the notes provided with each section as a quick review of these topics from Precalculus. Feel free to research these topics further if you need more information to successfully complete this packet. Your teacher will expect you to be comfortable with this content as we start Calculus. You will be given a test on these topics the first week of school so review these concepts carefully and devote the necessary time to make sure that you are well prepared for the coming course.

We look forward to working with you for the coming year!

Sincerely,
Miss Cathy Martin
Dr. Johnson Mathew

QUADRATIC EQUATIONS

A **quadratic equation** is always written in the form of:

$$ax^2 + bx + c = 0 \qquad \text{where} \quad a \neq 0$$

The form $ax^2 + bx + c = 0$ is called the *standard form* of a quadratic equation.

You may solve a quadratic equation using one of four methods:

The Square Root Property:

When $x^2 = a$, where a is a real number, then your $x = \pm\sqrt{a}$

FACTORING

Basic rules of factoring:

$a^2 + 2ab + b^2 = (a+b)^2$
$a^2 - 2ab + b^2 = (a-b)^2$
$a^2 - b^2 = (a+b)(a-b)$

$$\begin{cases} a^3 + b^3 = (a+b)(a^2 - ab + b^2) \\ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \end{cases}$$

If one of these rules does not work, consider factoring by decomposition.
Remember to always check your solutions. You can use direct substitution of the solutions in the equation to see if the solutions satisfy the equation.

Quadratic Formula

Factoring is useful only for those quadratic equations which have whole numbers. When you encounter quadratic equations that cannot be easily factored, use the quadratic formula to find the value of x:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the Square

1) If the leading coefficient is not 1, use the multiplication (or division) property of equality to make it 1
2) Rewrite the equation by sending the constant to the right side of the equation
3) Divide the numerical coefficient the middle term by 2, then square it, and add it to both sides of the equation, but leave the square form on the left side of the equation
4) Once you found the squared number rewrite the equation
5) Use the square root property to clear the term

Example:

$2x^2 + 4x - 16 = 0$
$x^2 + 2x - 8 = 0$
$x^2 + 2x = 8$
$x^2 + 2x + 1 = 8 + 1$

$(x + 1)^2 = 9$
$x + 1 = \pm 3$
$x = -1 \pm 3$
$x = -4$ or 2

QUADRATIC EQUATIONS – EXERCISES

Factor each of the following completely:

1. $9x^2 + 12x + 4$

2. $25x^2 - 9$

3. $27x^3 + 216$

4. $4x^2 - 12x + 9$

5. $x^3 - 64$

Solve by completing the square:

6. $x^2 - 2x = 8$

7. $x^2 - 4x + 12 = 0$

8. $3x^2 - 6x + 5 = 0$

9. $4x^2 - 12x + 8 = 0$

10. $3x^2 - 4x + 7 = 0$

Solve by the quadratic formula:

11. $3x^2 - 5x - 12 = 0$

12. $6x^2 + 9x - 6 = 0$

13. $9x^2 + 6x - 12 = 0$

14. $6x^2 - 13x = -6$

IMPORTANT NOTES ON GRAPHING WITH TRANSFORMATIONS

1. Always identify the ***basic function*** first. Your main choices are:

Basic Function	**Basic Equation**	**Basic Graph**
a line	$y = x$	***line***
a squaring function	$y = x^2$	***parabola "U"***
a cubing function	$y = x^3$	***"squiggly"***
a square root function	$y = \sqrt{x}$	***top of a sideways "U"***
an absolute value function	$y = \lvert x \rvert$	***"V"***
a reciprocal function	$y = \frac{1}{x}$	***hyperbola***

2. Consider ***reflections*** of $y = f(x)$.
   a) The graph of $y = - f(x)$ is the reflection of the graph of f in the x-axis
   b) The graph of $y = f\,(- x)$ is the reflection of the graph of f in the y-axis
   c) The graph of $y = f^{-1}(x)$ is the reflection of the graph of f over the line $y = x$

3. Consider ***translations*** $y = f(x)$.
   The graph of $\boldsymbol{y = f(x + c) + d}$ is a *vertical* and *horizontal* translation of the graph of $\boldsymbol{y = f(x)}$.

   Vertical Shift:
   a) If d is < 0, the graph shifts d units down
   b) If d is > 0, the graph shifts d units up

   Horizontal Shift:
   a) If c is < 0, the graph shifts c units to the right
   b) If c is > 0, the graph shifts c units to the left
   **** Remember *horizontal* will do the opposite of what you would "expect".****

4. Consider **dilations** of $y = f(x)$.
   The graph of $\boldsymbol{y = a\,f(x)}$ is a ***vertical dilation*** of the graph of $y = f(x)$.
   a) If $0< |a| <1$, the graph moves toward the x-axis and is a vertical shrink.
   b) If $|a| >1$, the graph moves away from the x-axis and is a vertical stretch.

   The graph of $\boldsymbol{y = f\,(ax)}$ is a ***horizontal dilation*** of the graph of $y = f(x)$.
   a) If $0< |a| <1$, the graph moves away from the y-axis and is a horizontal stretch.
   b) If $|a| >1$, the graph moves toward the y-axis and is a horizontal shrink.

Suppose $f(x) = x^2$
Describe the translation that occurs from $f(x)$ to $h(x)$.

15. $h(x) = x^2 - 1$ ______________________________

16. $h(x) = x^2 + 1$ ______________________________

17. $h(x) = (x - 3)^2 - 1$ ______________________________

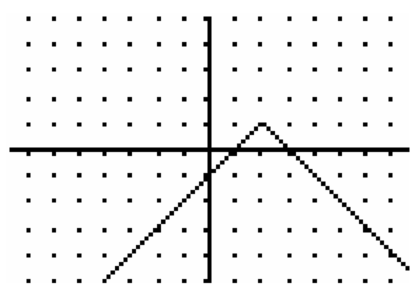
18. $h(x) = (-x + 3)^2$ ______________________________

19. $h(x) = - x^2$ ______________________________

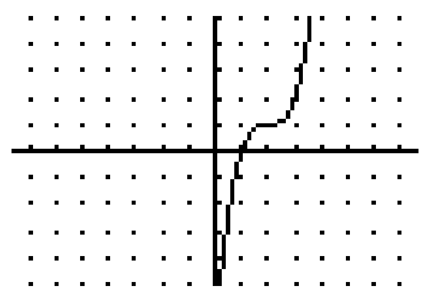
20. What is the domain $f(x) = \sqrt{x - 5}$? ______________________________

21. What is the range of $f(x) = x^2 - 2$? ______________________________

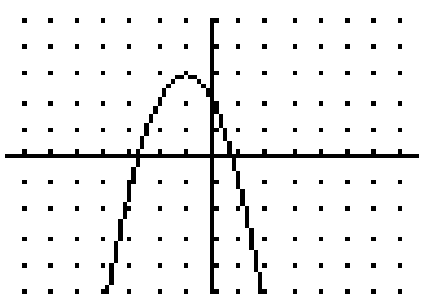
22. Identify the function for the graph shown.



A) $f(x) =$



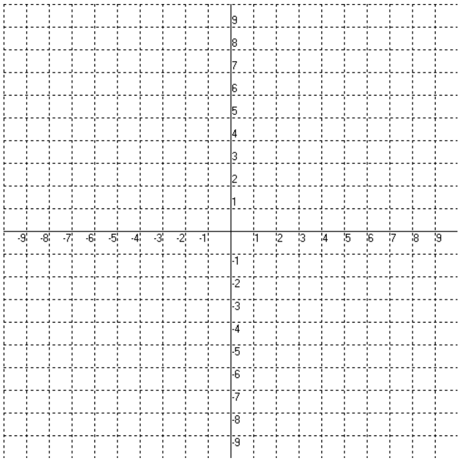
B) $f(x) =$



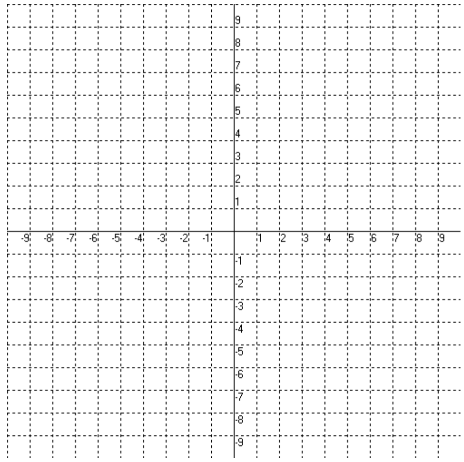
C. $f(x) =$

GRAPH EACH OF THE FOLLOWING WITHOUT USING A CALCULATOR.

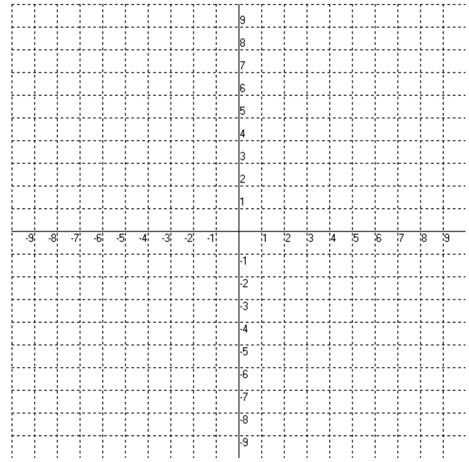
23. $f(x) = \dfrac{1}{x+2}$



24. $f(x) = 3x^3 - 8$

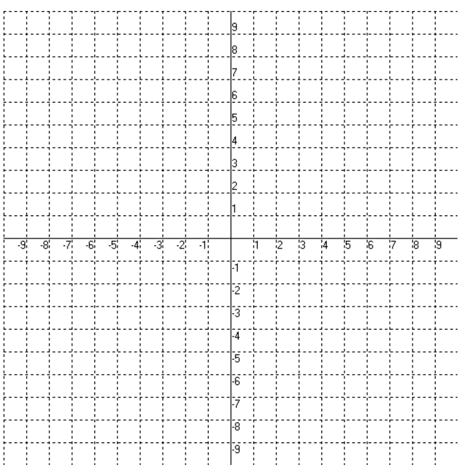


25. $f(x) = -3\,|x-2|$



26. Graph the following equation on the grid provided.

$$f(x) = -2(x+1)^2 - 3$$



A) What is the domain of $f(x)$?

B) What is the range of $f(x)$?

C) Over what intervals is the graph increasing?

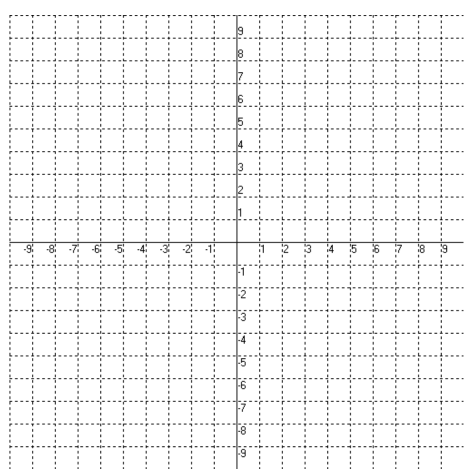
D) Over what intervals is the graph decreasing?

E) What are the relative maximums or minimums?

F) What is the y – intercept?

27. Graph the following equation on the grid provided.

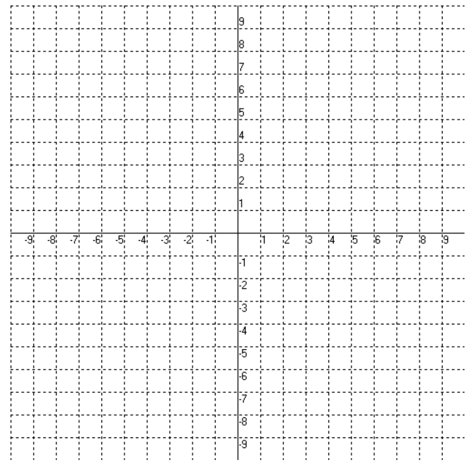
$$g(x) = \begin{cases} x+2, & x < 0 \\ x^2+2, & 0 \le x < 2 \\ x^3-1, & x \ge 2 \end{cases}$$



A) What is the domain of $g(x)$

B) What is the value of $g(2)$

28. Graph the following equation on the grid provided.

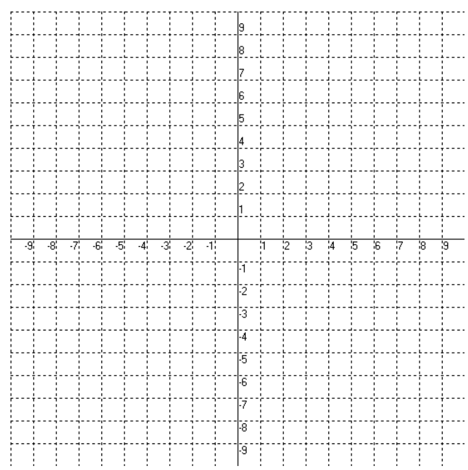


$$f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \le x \le 1 \\ x^2, & x > 1 \end{cases}$$

A) What is the domain of $f(x)$?

B) What is the value of $f(2)$?

29. Graph the following equation on the grid provided.

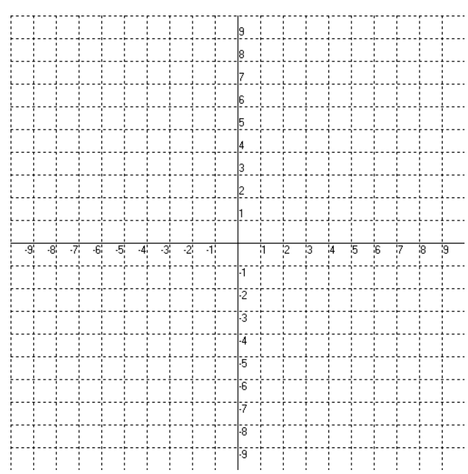


$$f(x) = \begin{cases} (x+2)^2 & x < -2 \\ \sqrt{2x+4} & x \ge -2 \end{cases}$$

A)What is the domain of $f(x)$?

B) What is the value of $f(2)$?

30. Graph the following equation on the grid provided.



$$f(x) = \begin{cases} 2x^3, & x < 0 \\ |x-2|, & 0 \le x \le 4 \\ 2x^2, & x > 4 \end{cases}$$

A)What is the domain of $f(x)$?

B) What is the value of $f(2)$?

COMBINATIONS OF FUNCTIONS

Sum, Difference, Product and Quotient of Functions

Let *f* and *g* be two functions with overlapping domains. Then, for all *x* common to both domains, the sum, difference, product and quotient of *f* and *g* are defined as follows:

Sum: $(f + g)(x) = f(x) + g(x)$

Difference: $(f - g)(x) = f(x) - g(x)$

Product: $(fg)(x) = f(x) \cdot g(x)$

Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \ g(x) \neq 0$

Use the following functions to answer each question.

$f(x) = \frac{x-4}{x^2-16}$ $\quad g(x) = 3x - 4$ $\quad h(x) = \sqrt{x^2 - 3}$ $\quad k(x) = x^2 - 4$

31. Find $g(x) + k(x)$

32. Find $\frac{f(x)}{g(x)}$

33. Find $k(x) - g(x)$

34. Find $g(x) \cdot k(x)$

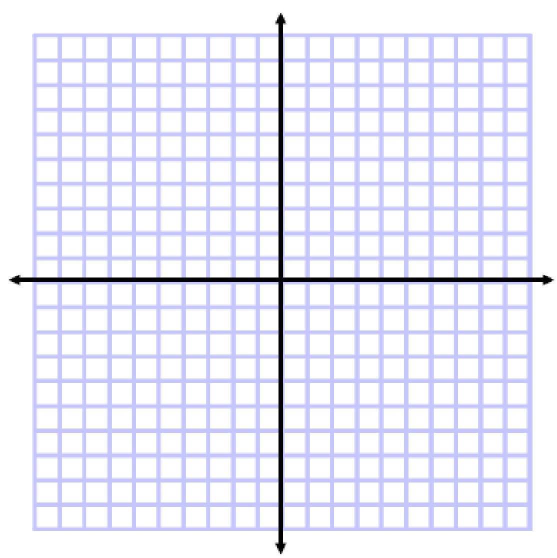
35. Find $(g \circ k)(x)$

36. Find $h(g(x))$

37. Find $\frac{k(x)}{g(x)}$

38. Find $g^{-1}(x)$

39. Graph $g(x)$, $g^{-1}(x)$, and $y = x$



40. Find $v^{-1}(x)$ if $v(x) = \sqrt{x - 4}$

Asymptotes of a Rational Function:

Let f be the rational function given by: $f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$

where $N(x)$ and $D(x)$ have no common factors.

The graph of f has a vertical asymptote at the zeros of $D(x)$.

The graph of f has one or no horizontal asymptote determined by comparing the degrees of $N(x)$ and $D(x)$.

- If $n < m$, (the degree of the numerator is less than the degree of the denominator) the graph of f has the line $y = 0$ (the x-axis) as a horizontal asymptote.
- If $n = m$, the graph of f has the line $y = \frac{a_n}{b_m}$ as a horizontal asymptote.
- If $n > m$, the graph of f has no horizontal asymptote.

41. Given the function: $f(x) = \frac{3x^2}{x^2 - x - 6}$

A. State the domain of the $f(x)$

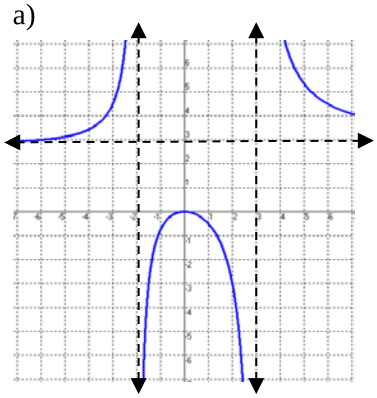
B. Find the vertical asymptotes of $f(x)$

C. Find the horizontal asymptotes of $f(x)$

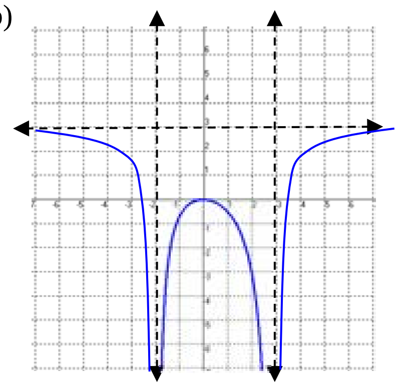
D. Find the zeros of $f(x)$

E. Which of the following is the graph of $f(x) = \frac{3x^2}{x^2 - x - 6}$?

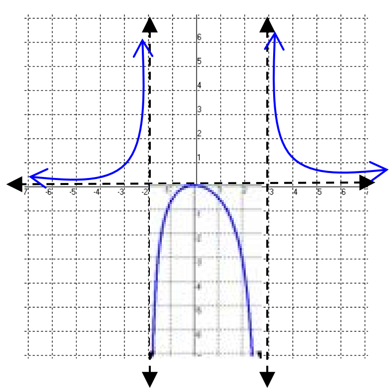
a)



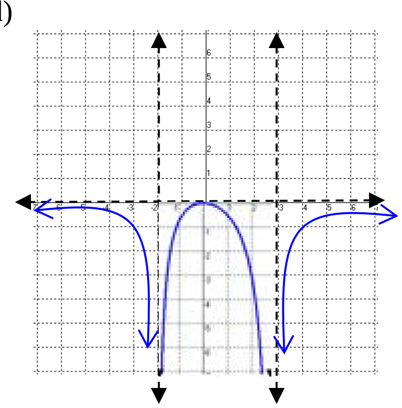
b)



c)



d)

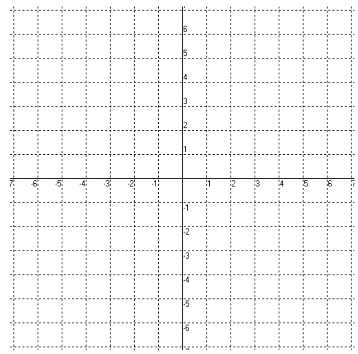


42. Given the function: $g(x) = \frac{3x}{x^2-5x+4}$

A. State the domain of the $g(x)$

B. Find the vertical asymptotes of $g(x)$

C. Find the horizontal asymptotes of $g(x)$



D. Find the zeros of $g(x)$

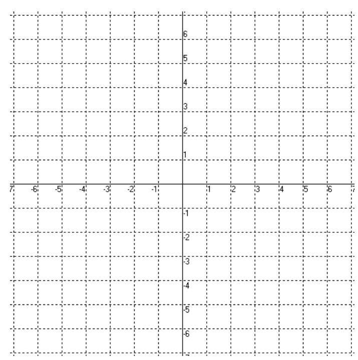
E. Sketch the graph of $g(x) = \frac{3x}{x^2-5x+4}$?

43. Given the function: $h(x) = \frac{x^2-8x+16}{x^2-2x-8}$

A. State the domain of the $g(x)$

B. Find the vertical asymptotes of. $g(x)$

C. Find the horizontal asymptotes of $g(x)$.



D. Find the zeros of $g(x)$

E. Sketch the graph of $h(x) = \frac{x^2-8x+16}{x^2-2x-8}$?

EXPONENTIAL FUNCTIONS AND LOGARITHMS:

DEFINITION OF EXPONENTIAL FUNCTION:
$f(x) = a^x$ where $a > 0, a \neq 1,$ and x is any real number.

Key points to plot: (0,1), (1, a) and (-1, $^1/_a$)
Important Properties:
$a^0 = 1$
$a^1 = a$
$a^x = a^y$ only if $x = y$ **Identity Property**

Log and Exponential equations are inverses.

DEFINITION OF LOGARITHMIC FUNCTION:
For $x > 0$ and $a > 0, a \neq 1, y = \log_a x$ iff $x = a^y$.

Key points to plot: (1,0), (a, 1) and ($^1/_a$,-1)
Important Properties:
$\log_a 1 = 0$
$\log_a a = 1$
$\log_a a^x = x$ and $a^{\log_a x} = x$
If $\log_a x = \log_a y$, then $x = y$. **Identity Property**
Natural log: $\ln x = \log_e x$

Change of base formula:

$\log_a x = \frac{\log_b x}{\log_b a}$

Properties of Logarithms:

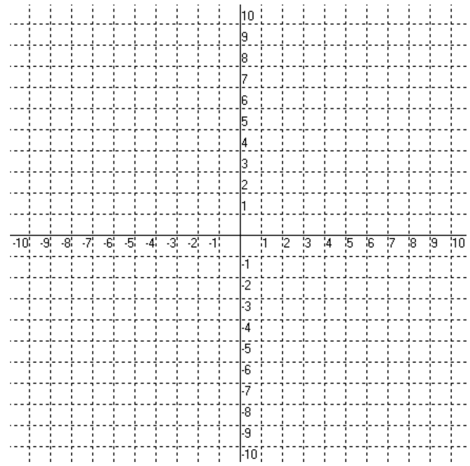
$$\log_a(uv) = \log_a u + \log_a v$$

$$\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$$

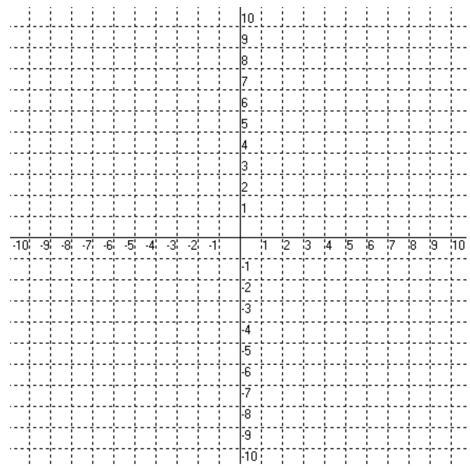
$$\log_a u^n = n \log_a u$$

Strategies for solving exponential and logarithmic equations:
1. Rewrite the equation in logarithmic form
2. Rewrite the equation in exponential form
3. Isolate the “ugly”….

44. Graph the equation $y = \log_4 (x) + 2$



45. Graph the equation $y = 6^{(x-4)}$



Solve each of the following equations:

46. $3\,(5^{x}) = 36$

47. $8\,(4^{2x}) - 16 = 40$

48. $\log_{6} x = \frac{3}{5}$

49. $\log_4 x - \log_4(x-1) = \frac{1}{2}$

50. $7 + 3 \ln x = 5$

51. $-14 + 3e^{x} = 11$

52. $\log_3 9 = x$

53. $\ln(x-2) = 8$

54. Expand: $\log_2 \frac{\sqrt{x}y^4}{z^4}$

55. Condense: $3\log_3 x + 4\log_3 y - 5\log_3 z$

UNDERSTANDING THE BASICS OF TRIGONOMETRY:

DEFINITIONS OF THE TRIG RATIOS:

$$\boldsymbol{\sin\theta = \frac{y}{r} = \frac{opposite}{hypotenuse}}$$

$$\boldsymbol{\cos\theta = \frac{x}{r} = \frac{adjacent}{hypotenuse}}$$

$$\boldsymbol{\tan\theta = \frac{y}{x} = \frac{opposite}{adjacent}}$$

$$\boldsymbol{\csc\theta = \frac{r}{y}}$$

$$\boldsymbol{\sec\theta = \frac{r}{x}}$$

$$\boldsymbol{\cot\theta = \frac{x}{y}}$$

Inverse or Arc is asking for what angle does the trig function have a given value. Restrictions apply on the domains/ ranges for inverses to stay functions.

$\sin^{-1}$ is for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$\cos^{-1}$ is for $0 \leq \theta \leq \pi$

$\tan^{-1}$ is for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

When solving equations with trig functions, be aware of the domain.

Are they asking for all possible solutions or are they asking for the solutions over a specific domain?

The restricted domain also indicates if the answer should be given in radian or degree form.

COMPLETE THE FOLLOWING TABLE OF VALUES:

TRIG FUNCTION	0^{o} ***or*** 0	30^{o} ***or*** _____	45^{o} ***or*** _____	60^{o} ***or*** _____	90^{o} ***or*** _____
$sin\ \theta$					
$cos\ \theta$					
$tan\ \theta$					

56. List the quadrants in which sine is positive. 56. ________________

57. List the quadrants in which cosine is positive. 57. ________________

58. List the quadrants in which tangent is positive. 58. ________________

59. In which quadrant is sine positive and cosine negative? 59. ________________

60. In which quadrant are both sine and tangent negative? 60. ________________

61. Given (8, -15) on a circle with center at the origin and a radius of r, forming an angle α, determine the value of the six trigonometric functions.

62. Given (-5, 12) on a circle with center at the origin and a radius of r, forming an angle β, determine the value of the six trigonometric functions.

61.	62.
sin α = _______	sin β = _______
cos α = _______	cos β = _______
tan α = _______	tan β = _______
csc α = _______	csc β = _______
sec α = _______	sec β = _______
cot α = _______	cot β = _______

Find the exact value of each of the following:

63. $\cos\left(\frac{10\pi}{3}\right)$

64. Sec$-\ \frac{\pi}{4}$

65. $tan\frac{3\pi}{2}$

66. $sin-\frac{7\pi}{4}$

67. $csc\frac{5\pi}{6}$

63. _______________

64. _______________

65. _______________

66. _______________

67. _______________

Solve for ***x*** on the interval $0 \leq x < 2\pi$ *(list all possible solutions*)

68. sin ***x*** = $\frac{\sqrt{2}}{2}$

69. cos ***x*** = $-\frac{1}{2}$

70. tan ***x*** = 0

71. cot ***x*** = $-\sqrt{3}$

72. sin ***x*** = – 4

68. _______________

69. _______________

70. _______________

71. _______________

72. _______________

Evaluate:

73. tan (arcsec 2)

74. csc $\left[arctan\frac{5}{8}\right]$

75. cos (*arc*cot2)

73. _______________

74. _______________

75. _______________

GRAPHING TRIG FUNCTIONS:

SINE: DOMAIN = ALL REAL NUMBERS; RANGE = d $\pm$ a

For the equation: y = a sin b(θ + c) + d *(Remember: sin 0 = 0)*

a = amplitude (always positive) ; period $=\frac{2\pi}{b}$; interval $= \frac{\boldsymbol{period}}{\mathbf{4}}$;

c = phase shift or horizontal shift; d = vertical shift

COSINE: DOMAIN = ALL REAL NUMBERS; RANGE = d $\pm$ a

For the equation: y = a cos b(θ + c) + d *(Remember: cos 0 = 1)*

a = amplitude (always positive) ; period $=\frac{2\pi}{b}$; interval $=\frac{period}{4}$;

c = phase shift or horizontal shift; d = vertical shift

TANGENT: DOMAIN HAS RESTRICTIONS ; RANGE = ALL REAL NUMBERS

For the equation: y = a tan b(θ + c) + d *(Remember: tan 0 = 0)*

a = how fast the graph moves away from the x axis ; **period** $=\frac{\pi}{b}$; interval $= \frac{period}{4}\ or\ \frac{period}{2}$

c = phase shift or horizontal shift; d = vertical shift

Use the equation below to determine each of the following:

76. $y = 6 \sin 4\left(\theta - \frac{\pi}{4}\right) + 1$

A. Horizontal shift and direction
B. Vertical shift and direction
C. Period
D. Amplitude
E. Domain
F. Range

77. $y = -4 \cos\left(3\theta + \frac{\pi}{6}\right) - 8$

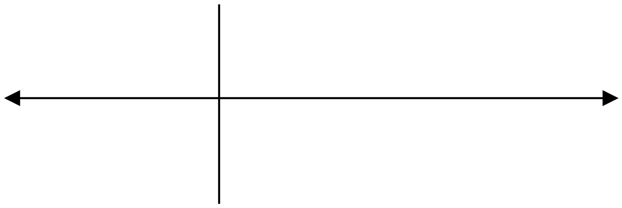
A. Horizontal shift and direction
B. Vertical shift and direction
C. Period
D. Amplitude
E. Domain
F. Range

78. $y = 3 \tan 2\,\theta$

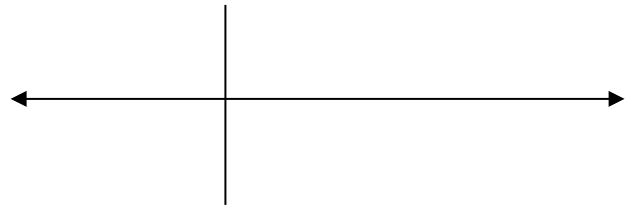
A. Horizontal shift and direction
B. Vertical shift and direction
C. Period
D. Equations of Asymptotes
E. Domain
F. Range

Graph each of the following: Label the vertical and horizontal axis

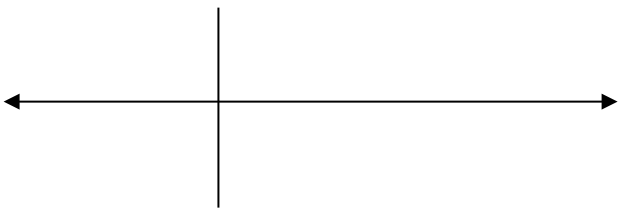
79. $y = 4 \sin \frac{3}{4} x$



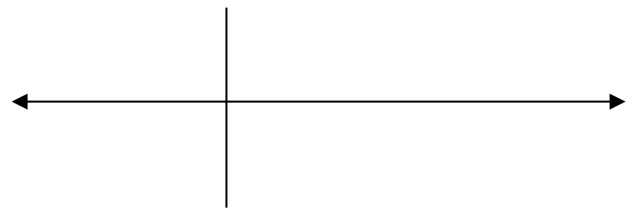
80. $y = -2 \cos\left(x - \frac{\pi}{3}\right)$



81. $y = 2 \tan\left(x + \frac{\pi}{3}\right)$

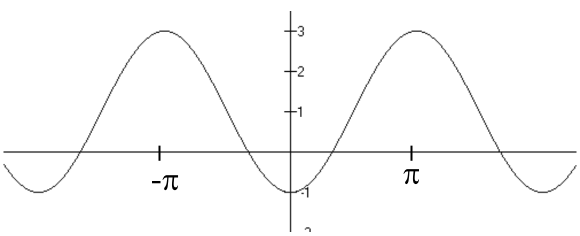


82. $y = -\ 4 \sin \theta + 2$



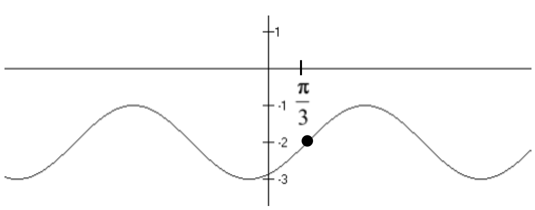
Write an equation for each of the following graphs.

83.



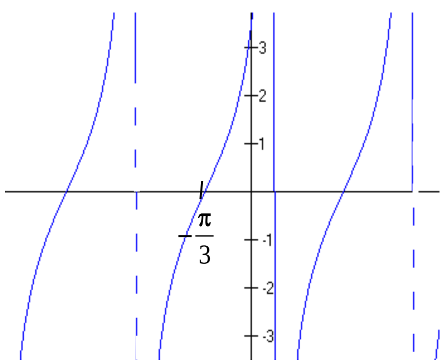
83. ______________________________

84.



84. ______________________________

85.



85. ______________________________

LAWS AND IDENTITIES:

RECIPROCAL IDENTITIES:

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

DOUBLE ANGLE FORMULAS:

$\sin 2u = 2\sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$

$\cos 2u = 2\cos^2 u - 1$

$\cos 2u = 1 - 2\sin^2 u$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

QUOTIENT IDENTITIES:

$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$

LAW OF SINES:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

OR

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

PYTHAGOREAN IDENTITIES:

$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$

LAW OF COSINES:

$c^2 = a^2 + b^2 - 2ab \cos C$

SUM AND DIFFERENCE IDENTITIES:

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

SIMPLIFY:

86. $\cos^2 x \sec^2 x - \cos^2 x$

87. $\frac{\sec x}{\csc x - \cot x} - \frac{\sec x}{\csc x + \cot x}$

88. $\sin\frac{\pi}{12}\cos\frac{11\pi}{12} - \sin\frac{11\pi}{12}\cos\frac{\pi}{12}$

89. $\cos u \ \cos\frac{\pi}{3} + \sin u \ \sin\frac{\pi}{3}$

VERIFY THE IDENTITY:

90. $\frac{\sin x}{1 - sin^2 \ x} = sex \ c \tan x$

91. $\sec x \sin^2 x + \cos x = \sec x$

SOLVE FOR X ON THE INTERVAL [0, 2π).

92. $5\sqrt{3}\tan x + 3 = 8\sqrt{3} \ \tan x$

93. $3 \ cot^2 x - 9 = 0$

94. $2\cos x \ \sin x + \sin x = 0$

95. $\tan^2\theta = -\frac{\sqrt{3}}{6} \ sec\theta$

FIND THE EXACT VALUE OF THE EXPRESSION:

96. cos 285

97. sin $\frac{5\pi}{12}$

FIND THE SIDE OR ANGLE NAMED.
GIVEN ΔABC.

98. $m\angle A = 40^o$; b = 10; a = 8. Find c.

99. $m\angle A = 40^o$; b = 10; c = 8. Find a.

100. Two cars leave the same spot and travel in different directions to form an angle (A) with their paths as its sides. The first car travels at a rate of 60 mph and the second car travels at a rate of 75 mph. If they continue along the straight paths for 2 hours and are 140 miles apart at the end of that time, what was the measure of angle A?