Name ________________________________ Date ____________________

Calculus 1- Summer Assignment

This assignment is an overview of the important concepts from Precalculus that will be used throughout the Calculus course. Students should work on not only completing the problems but also on making sure that they have a strong understanding of the content being applied in the solutions.

Instructions:
This assignment is due the first day of school.

- The problems are to be worked out neatly, in pencil on loose-leaf with all necessary work shown.
- Reviews of the important concepts are located in the boxes throughout the assignment.
- Students will be tested over this material at the beginning of the school year.
- This assignment will be counted as your first "group test" and will be graded for accuracy.
- Feel free to collaborate with your classmates—but you are responsible for handing in your *own* work.

Part 1: Functions and Graphing

For any function $f(x)$
Domain: the set of all possible values of x
Range: the set of all values of $f(x)$ or y
Domain restrictions: most ***restrictions*** for the domain are results of:

- denominators which cannot equal zero
- radicands that cannot be negative

Leading coefficient test:
Used to determine the end behavior of a polynomial function

degree of the polynomial is n	n is Even	n is Odd
Leading Coefficient is Positive	Left → ∞ Right → ∞	Left → - ∞ Right → ∞
Leading Coefficient is Negative	Left → - ∞ Right→ - ∞	Left → ∞ Right→ - ∞

Test for Even and Odd functions:
The function $y = f(x)$ is even if $f(-x)= f(x)$
Even functions are symmetric with the y-axis
The function $y = f(x)$ is odd if $f(-x)= - f(x)$
Odd functions are symmetric with the origin.

Basic Types of Transformations:
original graph: $y = f(x)$
Horizontal shift c units right: $y = f(x - c)$
Horizontal shift c units left: $y = f(x + c)$
Vertical shift c units downward: $y = f(x) - c$
Vertical shift c units upward: $y = f(x) + c$
Reflection over the x-axis: $y = - f(x)$
Reflection over the y-axis: $y = f(-x)$
Reflection over the origin: $y = -f(-x)$

Asymptotes and Holes:
Vertical: Occur when the function is undefined at an x value that cannot be canceled.

Example: If $f(x) = \frac{4x-6}{x^2-4}$,
Then $f(x)$ has vertical asymptotes at $x = \pm 2$

Hole: If a factor that causes a domain restriction is canceled, the graph will have a hole instead of an asymptote at that value of x.

Example: If $f(x) = \frac{3x+6}{x^2-4} = \frac{3(x+2)}{(x+2)(x-2)}$,

Then $f(x)$ has a hole at $x = -2$ and a vertical asymptote at $x = 2$.

Horizontal: If the degree of the numerator is equal to the degree of the denominator the horizontal asymptote is equal to the ratio of the leading coefficients

Example: If $f(x) = \frac{6x^3+3x^2-2}{5-4x^3}$, then $f(x)$ has a horizontal asymptote of $y = -\frac{6}{4}$

If the degree of the numerator is less than the degree of the denominator the asymptote is the x-axis

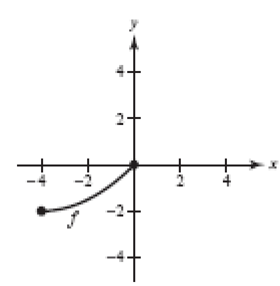
Example: If $f(x) = \frac{3x^2-2}{5-4x^3}$, then $f(x)$ has a horizontal asymptote at $y = 0$.

Slant: If the degree of the numerator is one more than the degree of the denominator the equation of the asymptote is the quotient of the function without the remainder.

1. Show that $y = \frac{x}{x^2+1}$ is symmetric with respect to the origin.

2. Given $f(x) = 3x - 7$, find $f(x + 1) + f(2)$

3. The domain of the function f shown in the figure to the right is $-4 < x < 4$. Complete the graph of f if f is odd.



4. Given the equation: $x^2 + y^2 + 4x - 6y + 12 = 0$ Solve for y.

Let $f(x) = x + \sqrt{x+2}$. Use a graphing calculator to graph $y = f(x)$

5. Estimate the domain and range from the graph

6. Use your calculator to determine the coordinates of any intercepts of the graph

7. Find the intercepts analytically.
8. Determine whether the function $f(x) = -x^4 + 2x^2 - 1$ is even, odd, or neither. Justify your answer.

9. For the functions $f(x) = x - 2$ and $g(x) = \frac{x+5}{3}$, find $g(f(x))$

Let $f(x) = \begin{cases} |x|, & x < 2 \\ x - 3, & x \geq 2 \end{cases}$. Evaluate each of the following:

10. $f(-3)$

11. $f(-2)$

12. $f(0)$

13. $f(2)$

A student working for a telemarketing company gets paid \$3 per hour plus \$1.50 for each sale. Let *x* represent the number of sales the student has in an 8-hour day.

14. Write a linear equation giving the day's salary S in terms of *x*.

15. Use the linear equation to calculate the student's salary on Wednesday if the student makes 14 sales that day.

16. Use a linear equation to calculate the number of sales per day the student would have to make in order to earn at least \$100 a day.

The table shows the average numbers of acres per farm in the US for selected years.

Year	1950	1960	1970	1980	1990	2000
Acreage	213	297	374	426	460	434

17. Plot the data where A is the acreage and t is the time in years, with t = 0 corresponding to 1950. Sketch a freehand curve that approximates the data.

18. Use the curve in part A to approximate A(15).

Find the domain, y-intercept (if it exists), and zeroes of each rational function.
Determine if the graph will have a hole or an asymptote at any undefined points.

19. $f(x) = \frac{4x}{x-3}$

20. $f(x) = \frac{-4x^2}{(x-2)(x+4)}$

21. $f(x) = \frac{3x(x-1)}{2x^2-5x-3}$

22. $f(x) = \frac{x-2}{x^3-8}$

23. $f(x) = \frac{-2(x^2-4)}{3(x^2+4x+4)}$

Find the vertical, horizontal and oblique (slant) asymptotes of each function.

24. $f(x) = \frac{3x}{x+4}$

25. $f(x) = \frac{x^3-8}{x^2-5x+6}$

26. $f(x) = \frac{x^3}{x^4-1}$

27. $f(x) = \frac{5-x^2}{3x^4}$

28. $f(x) = \frac{3x^4+4}{x^3+3x}$

29. $f(x) = \frac{x^3-1}{x-x^2}$

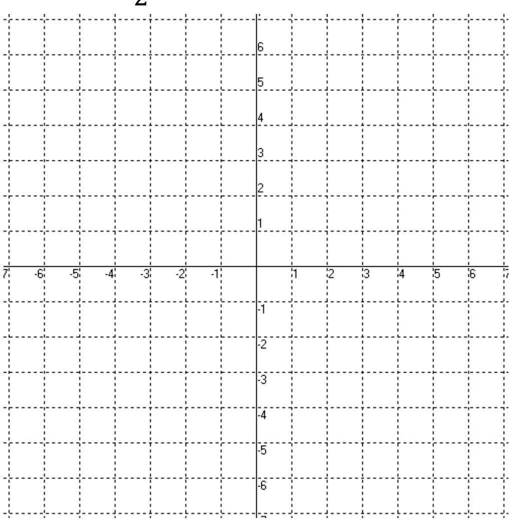
Without the use of a calculator, sketch the graph of each function on a separate grid.

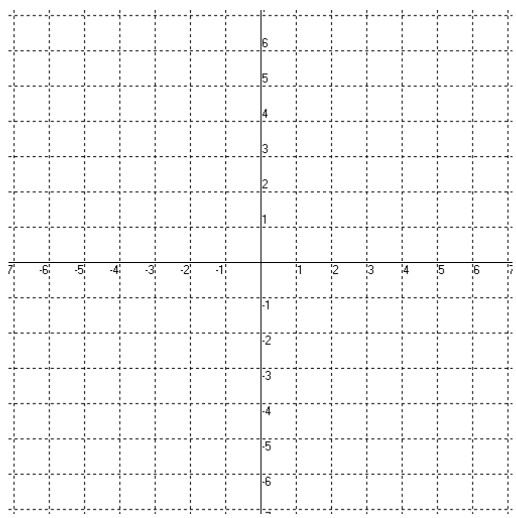
30. $y = \frac{3}{2}x + 1$

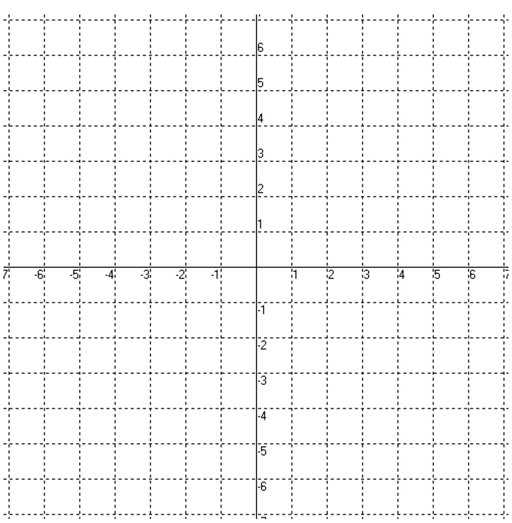
31. $y = (x - 3)^2$

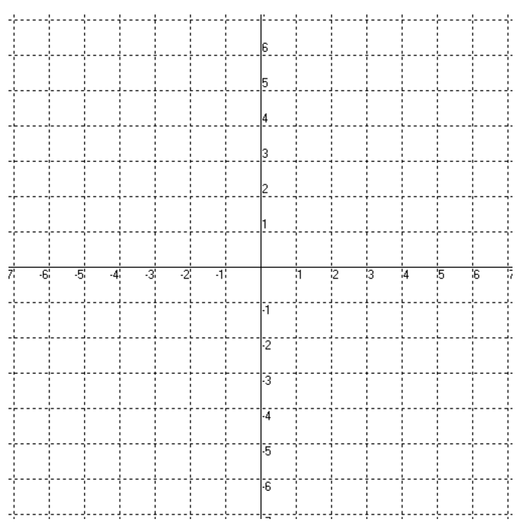
32. $y = |x + 3| - 1$

33. $y = \frac{3(x-2)}{x^2-4}$









Part 2: GRAPHING TRIGONOMETRIC FUNCTIONS:

Sine: Domain = All real numbers; Range = d + a

For the equation: y = a sin b(θ + c) + d *(Remember: sin 0 = 0)*

a = amplitude (always positive) ; period = $\frac{2\pi}{b}$; interval = $\frac{\boldsymbol{period}}{4}$;

c = phase shift or horizontal shift; d = vertical shift

Cosine: Domain = All real numbers; Range = d $\pm$ a

For the equation: y = a cos b(θ + c) + d *(Remember: cos 0 = 1)*

a = amplitude (always positive) ; period = $\frac{2\pi}{b}$; interval = $\frac{\boldsymbol{period}}{4}$;;

c = phase shift or horizontal shift; d = vertical shift

Tangent: Domain has restrictions ; Range = All real numbers

For the equation: y = a tan b(θ + c) + d *(Remember: tan 0 = 0)*

a = how fast the graph moves away from the x axis ; **period** = $\frac{\pi}{b}$; interval = $\frac{\boldsymbol{period}}{4}$ *or* $\frac{\boldsymbol{period}}{2}$ **;**

c = phase shift or horizontal shift; d = vertical shift

Use the equation below to determine each of the following: (See example on next page for help.)

$$y = 4\,\boldsymbol{cos}\,3(\theta - \frac{\pi}{4}) + 1$$

34. Horizontal shift and Vertical shift with direction
35. Period and Amplitude
36. Domain and Range

Use the equation below to determine each of the following:

$$y = -2\,\boldsymbol{sin}\,(2\theta + \frac{\pi}{6}) - 5$$

37. Horizontal shift and Vertical shift with direction
38. Period and Amplitude
39. Domain and Range

Use the equation below to determine each of the following:

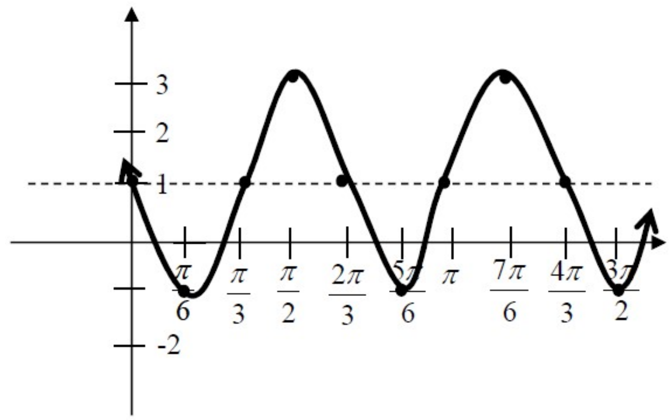
y = 3 ***tan*** 4*θ*

40. Horizontal shift and Vertical shift with direction
41. Period and Amplitude
42. Domain and Range

Graph at least two complete periods for each of the following without the use of a calculator.

Label the vertical and horizontal axis and clearly mark and label the relative maximums, relative minimums, and "zeros".

Example: $y = 2\sin 3(x - \frac{\pi}{3}) + 1$



A= 2, (amplitude is 2)

B= 3 (period is $\frac{\pi}{6}$)

C = $\frac{\pi}{3}$ (horizontal shift of $\frac{\pi}{3}$ to the right)

D= 1 (vertical shift of 1 up)

Domain is all real numbers. The range is $\{y: -1 \le y \le 3\}$

43. $y = 2\,sin\,\frac{1}{4}\,x$

44. $y = -3\cos(x - \frac{\pi}{6})$

45. $y = \mathbf{tan}\left(\frac{1}{2}x + \frac{\pi}{3}\right)$

46. $y = -4\mathbf{sin}\,(\boldsymbol{\theta})$

Graph each of the following without the use of a calculator. (Be sure to label the axis values!)

47. $y = \sin^{-1}(x)$

48. $y = tan^{-1}(x) + 2$

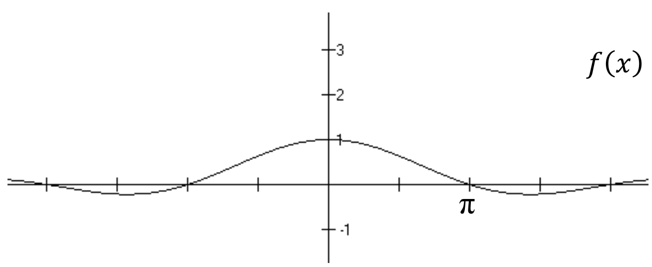
Consider the function $f(x) = \dfrac{\sin x}{x}$ and its graph shown in the figure.

49. What is the domain of the function?

50. Identify any symmetry and asymptotes of the graph.

51. Describe the behavior of the function as $x \to 0$.

52. How many solutions does the equation $\dfrac{\sin x}{x} = 0$ have in the interval [-8,8]? Find the solutions.



$f(x) = \dfrac{\sin x}{x}$

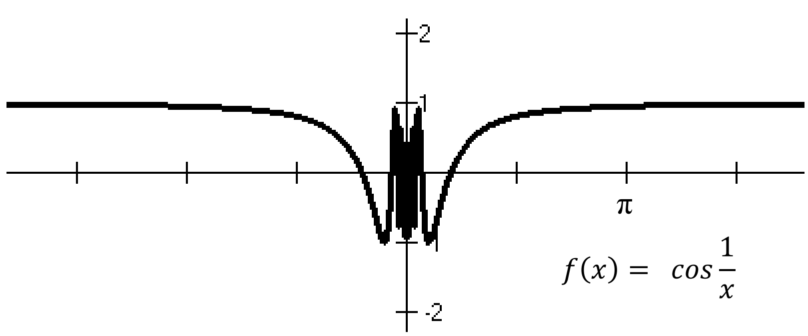
Consider the function $f(x) = cos\ \dfrac{1}{x}$ and its graph shown in the figure.

53. What is the domain of the function?

54. Identify any symmetry and asymptotes of the graph.

55. Describe the behavior of the function as $x \to 0$.

56. Does the equation $cos\ \dfrac{1}{x} = 0$ have a greatest solution? If so, approximate the solution. If not, explain why.



Part 3: Trigonometric Identities and Equations

Reciprocal Identities:

$sin\, u = \dfrac{1}{csc\, u}$ $cos\, u = \dfrac{1}{sec\, u}$ $tan\, u = \dfrac{1}{cot\, u}$

$csc\, u = \dfrac{1}{sin\, u}$ $sec\, u = \dfrac{1}{cos\, u}$ $cot\, u = \dfrac{1}{tan\, u}$

Double Angle Formulas:

$sin\, 2u = 2sin\, u\ cos\, u$

$cos\, 2u = cos^2 u - sin^2 u$

$cos\, 2u = 2cos^2 u - 1$

$cos\, 2u = 1 - 2sin^2 u$

$tan\, 2u = \dfrac{2\, tan\, u}{1 - tan^2 u}$

Quotient Identities:

$tan\, u = \dfrac{sin\, u}{cos\, u}$ $cot\, u = \dfrac{cos\, u}{sin\, u}$

Sum and Difference Identities:

$sin(u \pm v) = sin\, u\ cos\, v \pm cos\, u\ sin\, v$

$cos(u \pm v) = cos\, u\ cos\, v \mp sin\, u\ sin\, v$

$tan(u \pm v) = \dfrac{tan\, u \pm tan\, v}{1 \mp tan\, u\, tan\, v}$

Pythagorean Identities:

$sin^2\theta + cos^2\theta = 1$

$1 + tan^2\theta = sec^2\theta$

$1 + cot^2\theta = csc^2\theta$

Verify each identity

57. $\csc\theta\ \sin\theta + cot^2\theta = csc^2\theta$

58. $\sin\theta + \sin\theta\, cot^2\theta = \csc\theta$

59. $cot\ \theta\ \tan\theta + tan^2\theta = sec^2\theta$

60. $\csc^2\theta - \cot^2\theta = 1$

61. $\cot\theta \sec\theta \sin\theta = 1$

62. $\sin\left(\dfrac{\pi}{6} + \theta\right) + \cos\left(\dfrac{\pi}{3} + \theta\right) = \cos\theta$

Solve each equation for $0 \leq \theta 2\pi$.

63. $\sin\theta + 2\sin\theta\cos\theta = 0$

64. $2\sin\theta - 4 = -2\sin\theta$

65. $2\cos^2\theta + \cos\theta - 1 = 0$

66. $\cos\theta - 2\sin\theta\cos\theta = 0$

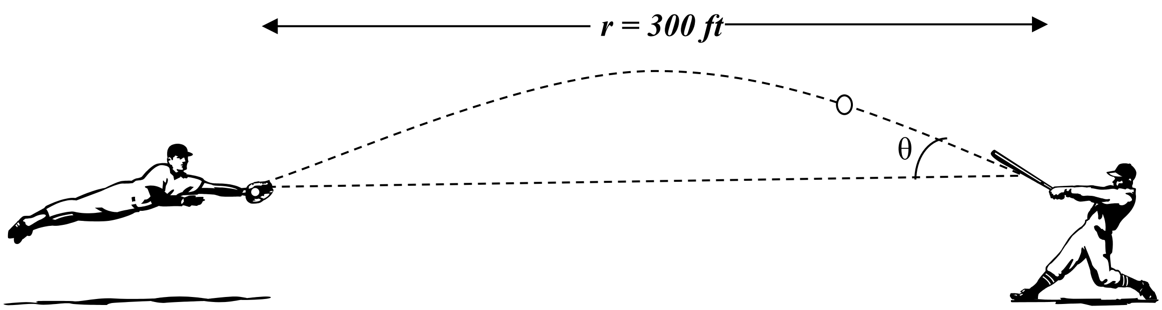
67. $2\sin^2\theta + 3\sin\theta = -1$

68. $\tan\theta(\sin\theta - 1) = 0$

69. $3\tan\theta = -\sqrt{3}$

70. $\sqrt{3} + 5\sin\theta = 3\sin\theta$

71. A batted baseball leaves the bat at an angle of θ with the horizontal and an initial velocity of $V_o = 100$ feet per second. The ball is caught by an outfielder 300 feet from home plate (see figure). Find θ if the range *r* of a projectile is $r = \frac{1}{32}v_0^2 \sin 2\theta$.



72. The monthly sales (in thousands of units) of a seasonal product are approximated by
$S = 74.50 + 43.75 \sin\frac{\pi t}{6}$ where *t* is the time in months, with *t* = 1 corresponding to January. Determine the months when sales exceed 100,000 units.

Part 4: Properties of Logs and *e*

LOGARITHMS

For $x > 0$ and $0 < a \neq 1$, $y = \log_a x$ *if and only if* $x = a^y$.

The function given by $f(x) = \log_a x$ is called the logarithmic function with base a.

The common log has an understood value of base 10. (When no base is given, then it is understood to have a base of 10.)

$f(x) = \log_e x = \ln x,\ x > 0$ is the natural log function.

EXPONENTIAL FUNCTIONS AND *e*

The exponential function f with base a is denoted by $f(x) = a^x$ where $a > 0$, $a \neq 1$, and x is any real number.

The number ***e*** is called the natural base.

e is a constant whose value is approximately 2.71828…

PROPERTIES OF LOGS:

$\log_a 1 = 0$ because $a^0 = 1$.

$\log_a a = 1$ because $a^1 = a$.

$\log_a(uv) = \log_a u + \log_a v$

$\log_a \frac{u}{v} = \log_a u - \log_a v$

$\log_a u^n = n \log_a u$

Inverse Property:

$\text{Log}_a a^x = x$ because in exponential form $a^x = a^x$.

One-to-one Property:

If $\log_a x = \log_a y$ then $x = y$

Properties of Natural Logarithms

$\ln 1 = 0$ because $e^0 = 1$

$\ln e = 1$ because $e^1 = e$

Inverse property: $\ln e^x = x$ because $e^{\ln x} = x$

One-to-One Property: If $\ln x = \ln y$, then $x = y$.

Change-of-Base Formula: $\log_a x = \frac{\log_b x}{\log_b a}$

Use the properties of logs and *e* to solve each equation. Approximate the result to three decimal places.

73. $e^{\ln x^2} + 10 = 46$

74. $1000e^{-4x} = 75$

75. $8(4^{6-2x}) + 13 = 41$

76. $e^{2x} - 5e^x + 6 = 0$

77. $e^{2x} + 9e^x + 36 = 0$

78. $\frac{400}{1+e^{-x}} = 350$

79. $\frac{119}{e^{6x}-14} = 7$

80. $\log 3z = 2$

81. $5 \log(x - 2) = 11$

82. $2 \ln e^x = 7$

83. $\ln \sqrt{x-8} = 5$

84. $\ln(x + 1) - \ln(x - 2) = \ln x$

85. $\log_2 x + \log_2(x + 2) = \log_2(x + 6)$

86. $\log_4 x - 4\log_4 2 = \log_4(x - 1)$

87. $\log 4x - \log(12 + \sqrt{x}) = 2$

Graphing: Graph each of the following equations without the use of a calculator. Clearly label the *x* and *y* intercepts and sketch any asymptotes on the graph.

88. $f(x) = 3^x$

89. $f(x) = \log_3 x$

90. $f(x) = \ln(x - 4)$

91. $f(x) = -e^{x+2}$

92. $f(x) = 2^{x-4} + 3$

93. $f(x) = \log_2 x - 1$

Applications:

The demand equation for a hand-held electronic organizer is $p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$

94. Find the demand *x* for a price of $p = \$600$ and $p = \$400$.

In a group project in learning theory, a mathematical model for the proportion *P* of correct responses after *n* trials was found to be $P = \frac{0.83}{1 + e^{-0.2n}}$. Use a graphing utility to graph the function.

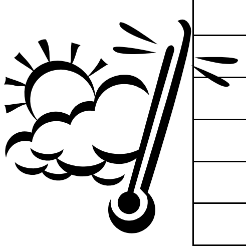
95. Use the graph to determine any horizontal asymptotes of the function. Interpret the meaning of the upper asymptote in the context of this problem.

96. After how many trials will 60% of the responses be correct?

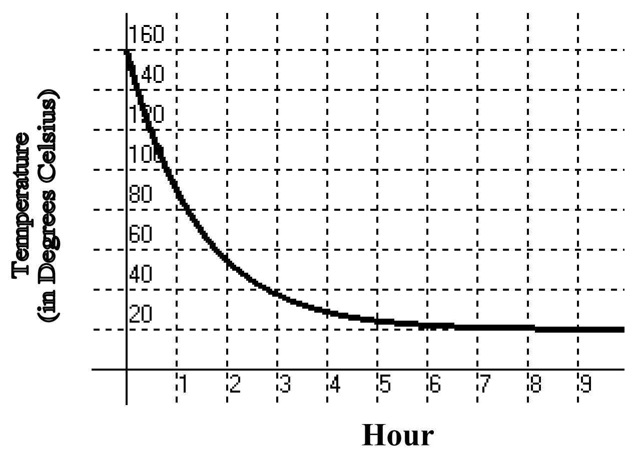
An object at a temperature of 160°C was removed from a furnace and placed in a room at 20°C. The temperature *T* of the object was measured each hour *h* and recorded in the table. A model for this data is $T = 20[1 + 7(2^{-h})]$. The graph of this model is shown in the figure.

97. Use the graph to identify the horizontal asymptote of the model and interpret the asymptote in the context of the problem.

98. Use the model to approximate the time when the temperature of the object was 100°C.



Hour, *h*	**Temperature, *T***
0	160°
1	90°
2	56°
3	38°
4	29°
5	24°



After discontinuing all advertising for a tool kit in 1998, the manufacturer noted that sales began to drop according to the model $S = \frac{500{,}000}{1 + 0.6e^{kt}}$ where S represents the number of units sold and $t = 0$ represents 1998. In 2000, the company sold 300,000 units.

99. Complete the model by solving for *k*.

100. Estimate sales in 2005.